On A Classification of Mesons

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Abstract

We give a mass formula for computing the mass spectrum of mesons. By this formula we show that there are many mesons with their masses corresponding to a prime number. In particular we show that all strange mesons are with their masses corresponding to a prime number. With these prime numbers indexing the mesons we give a classification of mesons. We set up a knot model of mesons to derive this mass formula. In this knot model mesons and their anti-particles are modeled by knots and their mirror images respectively. Then the amphichiral knots which are equivalent to their mirror images are used to model mesons which are identical with their anti-particles. With this knot model we show that there is a periodic phenomenon in the classification of mesons such that the starting nonet and the ending nonet are nonets of pseudoscalar mesons with the π meson modeled by an amphichiral knot. From this periodic phenomenon we give a theoretical argument for the existence of charm-anticharm mesons.

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We give a formula to compute the mass spectrum of mesons, as follows [1]. Let M denote the mass of a meson. Then we propose the following formula for M:

$$M = a \times bMev \tag{1}$$

where a and b are integers such that a is as a consecutive number for indexing the meson; b is as a number related to the total angular momentum J of the meson. This number b is proportional to the winding number of a knot model of the meson while a is as an index of the knot model of the meson [1]. The number b is around a number which is determined to be the number 45. As examples let us consider the nonet of pseudoscalar mesons. By using (1) we have that $\pi(135)$ is with the mass $3 \times 45 = 135Mev$ where $a = 3$ is as a consecutive number and $b = 45$ is as a number proportional to the winding number of a knot model of the meson $\pi(135)$. The number 45 is related to the total angular momentum J of $\pi(135)$. Similarly by using (1) we have that the masses of $\eta(549)$ and $\eta^{'}(958)$ are given by $13 \times 42 = 546Mev$ and $23 \times 42 = 966Mev$ respectively where $a = 13$ and $a = 23$ are as indexes for the knot models of $\eta(549)$ and $\eta^{'}(958)$ respectively.

Let us then consider strange mesons. For strange mesons let us modify (1) by introducing a number for the strange degree of freedom. We consider the following mass formula of a strange meson:

$$M = a \times b + n \times sMev \tag{2}$$

where the number s equals 12 or 24 is for the strange degree of freedom of the strange meson; n is an integer as the excited level of state of the strange meson and is also related to total angular momentum J of the strange meson. As an example let us consider the strange meson $K(498) = K^0(498)$ in the nonet of pseudoscalar mesons. By using (2) we have that the mass of $K(498)$ is given by $11 \times 43 + 1 \times 24 = 497Mev$ where $a = 11$, $b = 43$, $n = 1$ and $s = 24$. The two numbers 43 and 1 approximately correspond to that $J = 0$ for $K(498)$. Here the number $n = 1$ gives the excited level of strange state of $K(498)$.

Let us then consider mesons of the form $s\overline{s}$ where s denotes the strange quark. For such mesons let us modify (1) by introducing two strange degrees of freedom by the following formula:

$$M = a \times b + n \times s\overline{s}Mev \tag{3}$$

mesons of the form $s\overline{s}$ and n is an integer as the excited level of strange-antistrange state of the $s\overline{s}$ meson and is also related to the total angular momentum J of the $s\overline{s}$ meson. As an example let us consider the $\phi(1020)$ meson which is of the form $s\overline{s}$. By using (2) we have that the mass of $\phi(1020)$ is given by $23 \times 43 + 1 \times 32 = 1021 Mev$ where $a = 23$, $b = 43$, $n = 1$ and $s\overline{s} = 32$. The number 43 and 1 approximately correspond to that $J = 1$ for $\phi(1020)$. Here we have that $32 = 2 \times 16$ and we use the multiples of 16 as the $s\overline{s}$ degree of freedom. We remark that this $s\overline{s}$ degree of freedom is a multiple of 16 and is not a multiple of 12 (or 24) where 12 (or 24) is for the strange degree of freedom. We shall explain this phenomenon in a more complete investigation of this knot model of mesons [1].

With the above three formulas we then compute the masses of the light mesons listed in [2][3]. We list the results of this computation in the following table which is as a classification of mesons.

$I=1$	$I=0$	Strange mesons $I=\frac{1}{2}$	$I=0$
$\pi(135)$ $\mathbf{3}\times 45=135$	$\eta(549)$ $\mathbf{13}\times 42=546$	$K(498)$ $\mathbf{11}\times 43+24=497$	$\eta'(958)$ $\mathbf{23}\times 42=966$
$\rho(770)$ $\mathbf{17}\times 45=765$	$\omega(783)$ $\mathbf{17}\times 46=783$	$K^*(896)$ $\mathbf{19}\times 46+24=898$	$\phi(1020)$ $\mathbf{23}\times 43+32=1020$
$b_1(1235)$ $27\times 45=1242$	$h_1(1170)$ $26\times 45=1170$	$K_1(1270)$ $\mathbf{29}\times 43+24=1271$	$h_1(1380)$ $30\times 46=1380$
$a_1(1260)$ $28\times 45=1260$	$f_1(1285)$ $28\times 46=1288$	$K_1(1400)$ $\mathbf{31}\times 44+36=1400$	$f_1(1420)$ $\mathbf{29}\times 49=1421$
$a_0(980)$ $\mathbf{23}\times 43=989$	$f_0(980)$ $\mathbf{23}\times 43=989$	$K_0^*(1430)$ $\mathbf{31}\times 45+36=1431$	$f_0(1370)$ $\mathbf{29}\times 47=1363$
$a_2(1320)$ $30\times 44=1320$	$f_2(1270)$ $\mathbf{29}\times 44=1276$	$K_2^*(1430)$ $\mathbf{31}\times 45+36=1431$	$f_2(1430)$ $\mathbf{31}\times 46=1426$
$a_2(1320)$ $\mathbf{31}\times 43=1333$	$f_2(1270)$ $\mathbf{31}\times 41=1271$	$K_2^*(1430)$ $\mathbf{31}\times 45+36=1431$	$f_2'(1525)$ $\mathbf{37}\times 41=1517$
$\rho(1450)$ $\mathbf{31}\times 47=1452$	$\omega(1420)$ $\mathbf{31}\times 46=1426$	$K^*(1410)$ $\mathbf{31}\times 45+12=1407$	$\phi(1680)$ $\mathbf{37}\times 45+16=1681$
$\pi(1300)$ $\mathbf{31}\times 42=1302$ $\mathbf{29}\times 45=1305$	$\eta(1295)$ $\mathbf{31}\times 42=1302$	$K(1460)$ $\mathbf{31}\times 46+36=1462$	$\eta(1440)$ $\eta_H:\mathbf{37}\times 40=1475$ $\eta_L:32\times 43=1408$
$X(1600)=\pi_2(1600)$ $34\times 47=1598$	$f_2(1565)$ $34\times 46=1564$	$K_2(1580)$ $\mathbf{37}\times 42+24=1578$	$f_2(1640)$ $\mathbf{41}\times 40=1640$
$\pi_1(1405)$ $32\times 44=1408$	$f_1(1510)$ $32\times 47=1504$	$K_1(1650)$ $\mathbf{37}\times 44+24=1652$	$X(1650)$ $36\times 46=1656$
$\rho(1700)$ $\mathbf{37}\times 46=1702$	$\omega(1600)$ $\mathbf{37}\times 43=1591$	$K^*(1680)$ $\mathbf{37}\times 45+12=1677$	$X(1910)=\phi(1910)$ $\mathbf{41}\times 45+2\cdot 32=1909$
$\pi_2(1670)$ $\mathbf{37}\times 45=1665$	$\eta_2(1645)$ $35\times 47=1645$	$K_2(1770)$ $\mathbf{37}\times 47+36=1775$	$\eta_2(1870)$ $39\times 48=1872$
$\rho_3(1690)$ $36\times 47=1692$	$\omega_3(1670)$ $\mathbf{37}\times 45=1665$	$K_3^*(1780)$ $\mathbf{37}\times 47+36=1775$	$\phi_3(1850)$ $\mathbf{41}\times 44+48=1852$
$X(1775)=\pi_2(1775)$ $\mathbf{37}\times 48=1776$	$f_2(1810)$ $\mathbf{37}\times 49=1813$	$K_2(1820)$ $\mathbf{37}\times 49+12=1825$	$f_2(1950)$ $39\times 50=1950$
$\pi(1800)$ $\mathbf{43}\times 42=1806$	$\eta(1760)$ $\mathbf{43}\times 41=1763$	$K(1830)$ $\mathbf{43}\times 42+24=1830$	$\eta(2225)$ $\mathbf{53}\times 42=2226$
$a_0(1450)$ $33\times 44=1452$	$f_0(1500)$ $33\times 45=1495$	$K_0^*(1950)$ $\mathbf{41}\times 44+24=1951$	$f_0(1710)$ $38\times 45=1710$
$X(2000)=a_2(2000)$ $\mathbf{41}\times 49=2009$	$f_2(2010)$ $\mathbf{41}\times 49=2009$	$K_2^*(1980)$ $\mathbf{41}\times 48+12=1980$	$f_2(2300)$ $\mathbf{47}\times 49=2303$
$a_4(2040)$ $40\times 51=2040$	$f_4(2050)$ $\mathbf{41}\times 50=2050$	$K_4^*(2045)$ $\mathbf{41}\times 49+36=2045$	$f_4(2300)$ $\mathbf{47}\times 49=2303$
$\pi_2(2100)$ $\mathbf{43}\times 49=2107$	$f_2(2150)$ $\mathbf{43}\times 50=2150$	$K_2(2250)$ $\mathbf{43}\times 52+12=2248$	$f_2(2340)$ $\mathbf{47}\times 50=2350$
$a_0(2020)$ $\mathbf{43}\times 47=2021$	$f_0(2020)$ $\mathbf{43}\times 47=2021$	$K_0^*?$ $\mathbf{47}\times 47+36=2245$	$f_0(2200)$ $44\times 50=2200$
$\rho(2150)$ $\mathbf{43}\times 50=2150$	$\omega(2145)$ $\mathbf{43}\times 50=2150$	$K^*?$ $\mathbf{43}\times 52+12=2248$	$\phi?$
$\rho_3(2250)$ $\mathbf{47}\times 48=2256$	$\omega_3(2250)$ $\mathbf{47}\times 48=2256$	$K_3(2320)$ $\mathbf{47}\times 49+12=2315$	$\phi_3?$
$\rho_5(2350)$ $\mathbf{47}\times 50=2350$	$X(2440)=\omega_5(2440)$ $\mathbf{47}\times 52=2444$	$K_5^*(2380)$ $\mathbf{47}\times 50+36=2386$	$X(2680)=\phi_5(2680)$ $\mathbf{53}\times 50+32=2682$
$\pi_4(2250)$ $\mathbf{47}\times 48=2256$	$f_J(2220)$ $\mathbf{47}\times 47=2209$	$K_4(2500)$ $\mathbf{53}\times 47+12=2503$	$X(2360)=f_4(2360)$ $49\times 48=2352$
$a_6(2450)$ $\mathbf{47}\times 52=2444$	$f_6(2510)$ $48\times 52=2496$	$K_6^*?$ $\mathbf{47}\times 53+12=2503$	$f_6?$
$X(2710)=\pi(2710)$ $\mathbf{59}\times 46=2714$	$X(2750)=\eta(2750)$ $\mathbf{61}\times 45=2745$	$K(3100)$ $\mathbf{67}\times 46+24=3106$	$X(3250)=\eta(3250)$ $\mathbf{71}\times 46=3266$

number a. Also many mesons are indexed by a prime number a. In this table the prime indexed number a is in bold face.

In this table we use the index number a and the total angular momentum J which is related to the number b and the number n for the excited level of strange and anti-strange quarks as the basic characteristics for the classification of mesons. Thus the index number a is together with the usual classification J^{PC} to give a classification of mesons. However we shall relax the classification with the index PC that we allow a classification of mesons with mixed PC. We use the number b and the number n to roughly determine the number J of the meson for classification.

Let us first consider the first and the second row of nonets of this table. These two nonets are classified by four prime numbers $3, 11, 13$ and 23 and three prime numbers $17, 19, 23$ respectively. The second nonet is classified by three consecutive prime numbers. The number 23 is jumped from the second nonet to the first nonet and this jumping is regarded as the characteristic of mesons with the notation $'$ such as the meson $\eta^{'}(958)$. The number 46 is greater than 43. This difference of winding numbers gives an estimate of J that if the first nonet is with $J = 0$ then the second nonet is with $J = 1$.

Then starting from the third row of the table we have that the classification is simpler in that each row of mesons is classified by at most two consecutive prime numbers except that for the three rows of scalar mesons $a_0(980), f_0(980), K_0^*(1430)$ and $f_0(1370)$; $a_0(1450), f_0(1500), K_0^*(1950)$ and $f_0(1710)$; $a_0(1450), f_0(1500), K_0^*(1950)$ and $f_0(1710)$; the three rows starting with $\pi(1800), X(2000)$ and $a_4(2040)$ respectively; and the last row. The last low is very interesting in that it is again a nonet of pseudoscalar mesons which is similar to the first nonet of pseudoscalar mesons in that the π mesons in these two nonets are modeled by an amphichiral knot. We shall later investigate this periodic phenomenon in more detail.

In the row of the scalar mesons $a_0(980), f_0(980), K_0^*(1430)$ and $f_0(1370)$ we classify scalar mesons by three consecutive prime numbers $23, 29$ and 31. A characteristic of this classification of scalar mesons is that the index 31 for the strange meson $K(1430)$ is greater than the index 29 for the scalar meson $f_0(1370)$ and this is different from that for the pseudoscalar and vector mesons of the first and second rows of the table.

We remark that the properties and classification of scalar mesons have been studied in detail. Experiments show that the scalar mesons $a_0(980), f_0(980)$ have some property of mesons of the form $s\overline{s}$ [2][3][4]-[19]. Here we show that $a_0(980), f_0(980)$ and $\eta^{'}(958)$ are indexed by the same prime number 23 as the $s\overline{s}$ meson $\phi(1020)$. This argrees with the experiments on the scalar mesons $a_0(980), f_0(980)$ [4]-[19].

Then the scalar mesons $a_0(1450), f_0(1500), K_0^*(1950)$ and $f_0(1710)$ are with indexes $33, 41, 38$ which are in the region of the three consecutive prime numbers $31, 37$ and 41 and we may regard these four mesons as classified by the three consecutive prime numbers $31, 37$ and 41. We note that the index for $K_0^*(1950)$ is the prime number 41 which is greater that the prime number 37 (or 38) and this classification thus agrees with the classification for the scalar mesons $a_0(980), f_0(980), K_0^*(1430)$ and $f_0(1370)$ which are classified by the three consecutive prime numbers $23, 29$ and 31.

In this classification table except the first and the last row we have that the indexes a for the mesons of the same row and of the first and the second columns are of the same number except in some cases where these two indexes are slightly different but are still in the region between two consecutive prime numbers. This is the basic characteristic of this classification of mesons. From the table we see that the largest such exceptional difference is the pair $\pi_2(1670)$ and $\eta_2(1645)$ which are indexed by 37 and 35 respectively. We note that the exceptional first and last rows are similar in that they are indexed by four prime numbers while all other rows are indexed by at most three consecutive prime numbers. This similarity of the first and the last row shows that this is a periodic phenomenon of the classification table.

Let us consider the three rows started with the π mesons $\pi_1(1405), \pi_2(2100)$ and $\pi(2250)$. These three rows are with mixed PC. From the first row of the classification table we see that the $\pi(135)$ meson is classified to a nonet with four prime numbers. Then these three rows are compressed with only two consecutive prime numbers and that the first two members of each of these three rows are of the same index number. It is for this characteristic that these three rows are formed as classification of mesons with π mesons as the $I = 1$ mesons. We shall later show that although these three mesons are denoted as a π meson they are not a real π meson as the $\pi(135)$ meson in are modeled by nonamphichiral knots. It is also for this reason that a mixed PC classification is allowed in the formation of the three nonets for these three π mesons.

Let us then consider the two rows started with the mesons $X(1600)$ and $X(1775)$ respectively. These two rows are similar to the three rows started with the π mesons $\pi_1(1405), \pi_2(2100)$ and $\pi(2250)$. Thus we predict that $X(1600)$ is the meson $\pi_2(1600)$ and $X(1775)$ is the meson $\pi_2(1775)$.

Similarly for the row started with $\pi_4(2250)$ we predict that the meson $X(2360)$ is the meson $f_4(2360)$ and that the meson $f_J(2220)$ is the meson $f_4(2220)$. For this meson $f_J(2220)$ we have the computational result $\mathbf{47} \times 47 = 2209$. Here we have $b = 47$ which is larger than 46 but is smaller than 50. From this rough estimate we have that the J of this meson is roughly between 2 and 4. This agrees with experiments on the J for this meson [2][3]. Here we put it to this row to determine that the J of this meson is 4.

For the row started with the vector meson $\rho(1700)$ we have the meson $X(1910)$. Here from our classification by the index a and the usual J^{PC} classification we predict that $X(1910)$ is the vector meson $\phi(1910)$. Our computation predicts that this meson is in $n = 2$ excited level of $s\overline{s}$.

For the row started with the meson $X(2000)$ we predict that this meson is the meson $a_2(2000)$.

Let us consider the eighth row of mesons containing the $\eta(1440)$ meson. Experiments show that there are two peaks $\eta_H(1475)$ and $\eta_L(1410)$ for the $\eta(1440)$ meson [20]-[30]. Here from our computation we have two results on the mass of $\pi(1300)$: $\mathbf{31} \times 42 = 1302Mev$ and $\mathbf{29} \times 45 = 1305Mev$. Thus for the two peaks $\eta_H(1475)$ and $\eta_L(1410)$ we may have two classifications $\pi(1302), \eta(1295), K(1460), \eta_H(1475)$ and $\pi(1305), \eta(1295), K(1460), \eta_L(1410)$.

Similarly for the meson $f_2(1270)$ in the sixth and seventh rows we have two computational results: $\mathbf{29} \times 44 = 1276Mev$ and $\mathbf{31} \times 41 = 1271Mev$. We let these two results correspond to the two mesons $f_2(1430)$ and $f_2^{'}(1525)$. This then gives two classifications: $a_2(1320)$, $f_2(1270)$, $K_2^*(1430)$, $f_2(1430)$ and $a_2(1320)$, $f_2(1270)$, $K_2^*(1430)$, $f_2^{'}(1525)$.

We remark that the above double phenomenon for the two peaks of $\eta(1440)$ and for the two mesons $f_2(1430)$ and $f_2^{'}(1525)$ may be due to the fact that the two prime numbers 29 and 31 are close to each other. This fact gives two computations for the mass of $f_2(1270)$ and for the mass of $\pi(1300)$ respectively. These two computations then give two indexes for $f_2(1270)$ and for $\pi(1300)$ and thus give two classifications for $f_2(1270)$ and for $\pi(1300)$ respectively.

Let us consider the row started with $\rho(2150)$. For this row we predict the existence of a K^* meson with mass approximately equal to $\mathbf{43} \times 52 + 12 = 2248Mev$. This K^* meson is close to the $K_2(2250)$ meson. Experiments show that there are various peaks in strange meson systems in the $2150 - 2260Mev$ region [2][3][31]-[35]. We thus predict that this K^* meson is in this region.

Similarly for the row started with the meson $a_0(2020)$ we predict that there is a K_0^* meson with mass approximately given by $\mathbf{47} \times 47 + 36 = 2245Mev$. This K_0^* is also close to the $K_2(2250)$ meson. We thus predict that this K_0^* meson is also in this $2150 - 2260Mev$ region of strange meson systems.

Similarly we consider the row started with $a_6(2450)$. For this row we predict the existence of a K_6^* meson with mass approximately equal to $\mathbf{47} \times 53 + 12 = 2503Mev$. We predict that this K_6^* meson is close to the $K_4(2500)$ meson.

Let us now consider the last row of the classification table. We show that this row gives a periodic phenomenon of classification of mesons in a sense as follows. First we have that this row is again classified with four prime numbers as the first row of the classification table (For this last row the four prime numbers are consecutive). Then let us show that why the $X(2710)$ meson is a π meson. To this end let us model mesons as knots, as follows.

First let us index the prime knots by prime numbers by the following table [40]:

prime knot	$\mathbf{3_1}$	$\mathbf{4_1}$	$\mathbf{5_1}$	$\mathbf{5_2}$	$\mathbf{6_1}$
prime number		3	5	7	11
prime knot	$\mathbf{6_2}$	$\mathbf{6_3}$	$\mathbf{7_1}$	$\mathbf{7_2}$	$\mathbf{7_3}$
prime number	13	17	19	23	29
prime knot	$\mathbf{7_4}$	$\mathbf{7_5}$	$\mathbf{7_6}$	$\mathbf{7_7}$	$\mathbf{8_1}$
prime integer	31	37	41	43	47
prime knot	$\mathbf{8_2}$	$\mathbf{8_3}$	$\mathbf{8_4}$	$\mathbf{8_5}$	$\mathbf{8_6}$
prime integer	53	59	61	67	71

where the prime knot $\mathbf{3_1}$ is assigned with the number 1. We have that the prime knots $\mathbf{4_1}, \mathbf{6_2}$ and amphichiral knot in that they are equivalent to their mirror images. It follows that these three knots are suitable to model elementary particles which are identical with their anti-particles (For those elementary particles which are not identical with their anti-particles we then model them by nonamphichiral knots in such a way that their anti-particles are modeled by the mirror images of these nonamphichiral knots). Thus we have that it is suitable to model $\pi(135)$ with the prime knot $\mathbf{4_1}$ and to model $\rho(770)$ and $\omega(783)$ with the prime knot $\mathbf{6_2}$ since these mesons are identifcal with their anti-particles. Now we have that the $K(3100)$ meson is assigned with the prime number 67 and since the prime number 59 is assigned to the amphichiral knot $\mathbf{8_3}$ we have that the meson $X(2710)$ should be a π meson since $X(2710)$ is indexed by 59 and thus we have $X(2710) = \pi(2710)$.

We remark that we may also use nonamphichiral knots to model elementary particles which are identical with their anti-particles by using an average of the nonamphichiral knot with its mirror image. Here the amphichiral knots $\mathbf{4_1}, \mathbf{6_2}$ and $\mathbf{8_3}$ are special in that they are only for modeling mesons which are identical with their anti-particles. Then for a meson such as the $\pi(135)$ meson which is modeled by the amphichiral knot $\mathbf{4_1}$ we have that this meson should have special property which is only from an amphichiral knot. For the $\pi(135)$ meson this special property is that it is the starting meson of the family of mesons. Then when the meson $X(2710)$ is identified as a special π meson modeled by the amphichiral knot $\mathbf{8_3}$ with the index number 59 it is then similar to the the starting $\pi(135)$ meson modeled by the amphichiral knots $\mathbf{4_1}$. This thus gives a periodic phenomenon of the classification of mesons.

It is then interesting to note from the classification table that the prime numbers 59 and 61 are not occupied with the K mesons while the consecutive prime numbers starting from 29 upto 53 are all occupied with the K mesons. This is similar to that the prime numbers $3, 5, 7$ are not occupied with the K mesons. Let us explain this phenomenon as follows. We have that 59 should not be occupied with the K mesons since 59 is assigned to the amphichiral knot $\mathbf{8_3}$ and the K mesons are not identified with their anti-particles. Then let us suppose that there is a K meson which occupies the prime number 61. Then this K meson should not be in a nonet with the $I = 1$ nonstrange meson assigned with the number 59 because the amphichiral knot $\mathbf{8_3}$ is for mesons such as the π meson with $I = 1$ having the property that the index number for the K meson does not consecutively follow 59 (This is similar to the amphichiral knot $\mathbf{4_1}$ for the meson $\pi(135)$). Thus in this nonet the meson with $I = 1$ is assigned with an index number less than 59. However we see from the classification table that starting from the second nonet the difference between the index number of meson with $I = 1$ and the index number for the K meson in the same nonet is not more than the difference between the two consecutive prime numbers where the larger one is the prime index number of the K meson. Thus according to this rule there will have no K mesons occupying the prime number 61.

Now since there are no K mesons occupy the prime numbers 59 and 61 this gives a room for introducing a new degree of freedom. We have that $\mathbf{61} \times 50 + 48 = 3098Mev$ where $48 = 2 \times 24$ is similar to 2×16 for the $\phi(1020)$ meson where 2×16 is for the two strange degrees of freedom of the $\phi(1020)$ meson. This approximates well the experimemtal mass $3096Mev$ of the J/ψ meson which is of the form $c\overline{c}$ where c denotes the charm quark. The number $48 = 2 \times 24$ is then for the two charm degrees of freedom of J/ψ and thus the prime integer 61 is assigned to the J/ψ meson. Then for the prime number 59 we have $\mathbf{59} \times 50 + 24 = 2974Mev$. This approximates the experimemtal mass $2979Mev$ of the $\eta_c(2979)$ meson. Thus $\eta_c(2979)$ is indexed by the prime number 59. The excited level of $c\overline{c}$ of $\eta_c(2979)$ is proportional to 24 which is half of 48 of the $c\overline{c}$-meson J/ψ. This agrees with the property of $\eta_c(2979)$ that it is partially a $c\overline{c}$-meson but is not a complete $c\overline{c}$-meson such as the J/ψ meson.

We note that since $\eta_c(2979)$ is indexed by the prime number 59 we have that $\eta_c(2979)$ is modeled by the amphichiral knot $\mathbf{8_3}$. This agrees with the usual classification that $\eta_c(2979)$ is usually classified to the beginning nonet $\pi(135), \eta(549), K(498)$ and $\eta^{'}(958)$. The reason for this agreement is that in this beginning nonet $\pi(135)$ is modeled by an amphichiral knot and it is as a periodic phenomenon that $\eta_c(2979)$ is also modeled by an amphichiral knot and according to this periodic phenomenon we have that these two mesons should be put into the same class.

Let us then continue to consider more $c\overline{c}$-mesons listed in [3]. We have the following table of

$c\bar{c}$-meson	computed mass
$\eta_c(1S)(2979)$	$\mathbf{59}\times 50+24=2974$
$J/\psi(1S)(3096)$	$\mathbf{61}\times 50+48=3098$
$\chi_{c0}(1P)((3415)$	$\mathbf{71}\times 48=3408$
$\chi_{c1}(1P)((3510)$	$\mathbf{73}\times 48=3504$
$h_c(1P)(3526)$	$\mathbf{73}\times 48+24=3528$
$\chi_{c2}(1P)((3556)$	$\mathbf{79}\times 45=3555$
$\eta_c(2S)(3594)$	$\mathbf{79}\times 45+36=3591$
$\psi(2S)(3686)$	$\mathbf{73}\times 50+36=3686$
$\psi(3770)$	$\mathbf{83}\times 45+36=3771$
$\psi(3836)$	$\mathbf{83}\times 45+2\cdot 48=3831$
$\psi(4040)$	$\mathbf{89}\times 45+36=4041$
$\psi(4160)$	$\mathbf{83}\times 50+2\cdot 36=4162$
$\psi(4415)$	$\mathbf{97}\times 45+48=4413$

where the numbers $12, 24, 36$ and 48 are for the $c\bar{c}$ degree of freedom.

From this table we see that the computed masses approximate well the experimentl masses. We notice that the computed masses of χ mesons do not have the term of multiple of the numbers $12, 24, 36$ and 48. This agrees with the property of χ mesons that they are not a complete $c\bar{c}$ meson. Also we notice that there are many $c\bar{c}$-mesons in this mass region. Let us give a knot model argument to explain this phenomenon. Let us continue the above table of indexing prime knots by prime integers with the following table:

prime knot	$\mathbf{8_7}$	$\mathbf{8_8}$	$\mathbf{8_9}$	$\mathbf{8_{10}}$	$\mathbf{8_{11}}$
prime number	73	79	83	89	97
prime knot	$\mathbf{8_{12}}$	$\mathbf{8_{13}}$	$\mathbf{8_{14}}$	$\mathbf{8_{15}}$	$\mathbf{8_{16}}$
prime number	101	103	107	109	113
prime knot	$\mathbf{8_{17}}$	$\mathbf{8_{18}}$			
prime integer	127	131			

In this table we list prime knots with eight crossings. In knot theory we have the fact that the sets of prime knots with even crossings such as $\mathbf{6}_{(\cdot)}, \mathbf{8}_{(\cdot)}, \mathbf{10}_{(\cdot)}$ contain amphichiral knots while the sets of prime knots with odd crossings such as $\mathbf{7}_{(\cdot)}, \mathbf{9}_{(\cdot)}$ contain no (or only a few) amphichiral knots. In this table of prime knots with eight crossings we have amphichiral knots $\mathbf{8_3}, \mathbf{8_9}, \mathbf{8_{12}}, \mathbf{8_{17}}, \mathbf{8_{18}}$ while there are no amphichiral knots of the form $\mathbf{7}_{(\cdot)}$ and $\mathbf{9}_{(\cdot)}$. Since these amphichiral knots are suitable for modeling mesons which are identical with their anti-particles such as the π and $c\bar{c}$ mesons this explains that why there is a family of $c\bar{c}$ mesons appearing in this mass region. Here we predict that there will have $c\bar{c}$ mesons modeled by the amphichiral knots $\mathbf{8_{12}}, \mathbf{8_{17}}, \mathbf{8_{18}}$ which are indexed by the prime numbers 101, 127 and 131 respectively.

To conclude, we have given a mass formula to compute the mass spectrum of mesons. This computation approximates well the experimental mass spectrum of mesons. We show that all strange mesons are indexed by a prime number. From these consecutive prime numbers we give a classification of mesons. These consecutive prime numbers are also used to index the prime knots. This then gives a modeling of mesons by knots. We show that the $\pi(135)$ meson with $I=1$ in the beginning nonet is modeled by an amphichiral knot $\mathbf{4_1}$. Then we show that the $I=1$ meson in the last nonet of light mesons is also modeled by an amphichiral knot which is $\mathbf{8_3}$ assigned with the prime index 59. We show that this is a periodic phenomenon from which a new charm-anticharm degree of freedom can be introduced. Then we show that the beginning charm-anticharm meson $\eta_c(2979)$ is also modeled by the amphichiral knot $\mathbf{8_3}$. This again shows the periodic phenomenon and is exactly in agreement with the usual classification of mesons that the $\eta_c(2979)$ meson is to be classified to the beginning nonet with $\pi(135)$ as the $I=1$ meson.

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